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The experiment was also made of crossing the spectra at some other angle than 0° or 180°. For instance, the rulings of the Ives grating were placed at right angles to those of the interferometer grating, as in Newton's method of crossed prisms. Seen in the telescope (adjusted for inverted spectra) the two spectra now made an elbow with each other, while the D_1 D_2 lines were still parallel and could be put in coincidence. On using a long (meter) collimator, strong interferences were obtained in the line of symmetry of the elbow and normal to the D lines. They have the same characteristics as the preceding and persist during a displacement of mirror of about 0.3 cm.

SEX INTERGRADES IN A SPECIES OF CRUSTACEA

By Arthur M. Banta

STATION FOR EXPERIMENTAL EVOLUTION, CARNEGIE INSTITUTION OF WASHINGTON

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In unisexual forms we are accustomed to think of the term male as signifying the possession by the animal indicated not only of a sperm-producing organ and the associated accessory male reproductive structures but also of certain definite secondary sex characters, definite characters of form and structure and, further, of conspicuous physiological and psychological characteristics. The term female implies definite and marked characters contrasting with those of a male on all these points. In other words maleness and femaleness are generally taken to indicate definite and precise opposed and alternative states, only one of which may obtain in a single individual.

The occurrence of known deviations from the conditions of maleness and femaleness are in the main confined to a comparatively few isolated and sporadic cases of hermaphroditism and gynandromorphism. Gynandromorphs are really sex mosaics inasmuch as a definite portion of the body, frequently one-half, possesses *in toto* the definite characters of one sex and the remainder of the body is distinctively of the other sex. However two workers have published concerning stocks in which considerable numbers of individuals are intermediate in their sexuality—intermediate not as sexual mosaics but quantitatively and as a whole different from either the normal male or the normal female. Riddle[1] has demonstrated intermediate sex forms in his pigeons and Goldschmidt has obtained such forms in the offspring of certain crosses in the gypsy moth.

In a species of crustacean the writer has obtained intermediate sex forms (for which the term *sex intergrade* seems appropriate) constituting a large percentage of the population. The significance of this material may better be considered after some acquaintance with it.

About four years ago two lines of a species of small phyllopod crustacean, *Simocephalus vetulus*, were taken from outdoor ponds, and their propagation continued in the laboratory. Throughout the experiment young have been isolated in individual bottles when not more than a day old. They reproduce parthenogenetically beginning when seven to twelve days of age, the first brood ordinarily containing from 8 to 20 young.

In addition to numerous other lines of this and other species of Cladocera, six separate strains of one line, '740,' have been propagated. For 130 generations these six strains (like all the other strains of Simocephalus to date) produced nothing but parthenogenetic females. Several lines of each of four other species of Cladocera have been propagated for long periods of time in the same manner as Simocephalus and except for a few males in one of the many lines they have produced only females.

But last October in the 131st generation of one of the strains of line 740 there suddenly appeared a large percentage of males together with normal females and a large number of sex intergrades—males with one or more female secondary sex characters, females with one to several male characters and some hermaphrodites with various combinations of male and female secondary sex characters.

At this point we may consider the criteria of sex in this species. The character of the sex gland,—whether a testis or an ovary—is readily determined by microscopic observation of the living animal. In addition, eight secondary sex characters distinguish the female from the male sex: (1) *body size*, the females rapidly outstrip the males in growth and are considerably larger at sexual maturity; (2) *size and position of eye*, the eye in the female is smaller than in the male and does not crowd the margin of the head as in the male; (3) *outline of the head*, the antero-ventral margin of the head is less angulated in the female than in the male; (4) *absence of the nuchal protuberance*, there is a slight dorsal protuberance immediately anterior to the cervical suture, typically in the male only; (5) and (6) *character of the first* (*rudimentary*) *antennae*, in the female the basal portion of the antenna is not enlarged and bears a single lateral spine (or stylet), whereas in the male the swollen condition of the basal portion of the antenna and the presence of two lateral stylets are distinctive. Since the two antennae in the same individual frequently are unlike, the character of each antenna counts as a separate diagnostic character;

(7) and (8) *outline and armature of the lateral post-abdomen margins,* in the female these margins are decidedly concave and bear posteriorly no teeth or only rudimentary 'teeth.' In the male the margin is slightly or not at all concave and bears larger teeth which also extend posteriorly much farther than in the female. Since the two post-abdominal margins often differ in character the nature of each margin constitutes a diagnostic sex character.

That these eight characters are really diagnostic secondary sex characters for the female is indicated by the fact that they occur in all individuals in the lines of Simocephalus which are reproducing parthenogenetically in the usual fashion, i.e., producing nothing but females. Again in a derived strain, in which for a time there appeared both females and males, but no sex intergrades, each female and male possessed all the secondary sex characters indicated above for its sex.

The sex intergrades are of almost all possible sorts from females with a single male secondary character to females with all eight of the secondary characters male, hermaphrodites with many combinations of secondary sex characters, and, finally males with one to several (never as many as eight, however) female characters.

The highly male-like female intergrades produce few young. Not infrequently they are entirely sterile. A female intergrade with as many as six strong male secondary characters rarely produces young. Many female intergrades with fewer than six male secondary sex characters are likewise sterile. In most of the sterile individuals eggs are produced but either they fail to develop after passing into the brood pouch or, more generally, the ovarian development is not completed and the eggs disintegrate in the ovary. To cite illustrative cases: two female intergrades in the 132d generation each had five male characters and though eggs began to form, the ovarian development was not completed and no eggs ever appeared in the brood pouch. A sibling of these had only the antennae male in character yet produced only a single brood. This case however is rather unusual as such individuals are usually quite prolific. In the 133d generation a female intergrade with four male secondary characters produced three broods consisting in all of only 15 individuals. On the other hand some individuals with several secondary male characters prove to be very prolific. A sibling of the last-mentioned individual had five secondary male characters and in addition one side of its post-abdomen was somewhat male in character yet it had a high prolificacy. In general in addition to being more prolific one may say that female intergrades with few o less distinctly male characters produce a smaller percentage of males and sex intergrades than those having a larger number or more definitely male characters.

Males that bear one or more female secondary sex characters in nearly every case have incompletely developed reproductive systems—one or both testes, though filled with sperm, being shorter than normal or lacking all or part of the sperm duct. In most cases the entire sperm duct is lacking. Some males, normal in their secondary sex characters, have similar incomplete reproductive systems.

By propagating from female intergrades we are able to continue the production of mixed broods—females, males and sex intergrades. This has now continued for 16 generations with no apparent change in the ratio of the various sex forms and with no apparent tendency of the stock to lose vigor or become less prolific.

From the sex intergrade stock we have derived in its third generation (the 133d generation under laboratory conditions) a normal strain producing nothing but typical females.

The origin of this all-female-producing strain is of interest. An almost sterile hermaphroditic intergrade produced only two young. One of these offspring was a female intergrade. This individual produced 202 young, 2 of which were normal females, 16 were female intergrades, 4 were hermaphrodites, 12 were male intergrades and 67 were normal males. In addition 3 individuals were not examined while 10 females (including female intergrades) and 88 males (including male intergrades) were undescribed. Probably only two individuals (1% of the whole), from the entire number of offspring from this individual were normal females. The only sibling of this female intergrade was a normal female which produced nothing but typical females, and from this individual came a derived normal strain which has now descended for 23 generations in the usual fashion without reversion to the production of sex intergrades. For two generations, however (the 13th and 14th from its origin from the sex intergrade strain), this derived strain did produce a considerable percentage of males. These males were all normal however and their sisters were all normal females, there being no return to the production of sex intergrades as in the parent stock. Following these two generations, in which males were produced, this derived strain has produced nothing but normal females in the nine generations which have occurred to date (August 10). From the 11th generation of the sex intergrade strain a second normal strain producing only typical females has been derived. More recently two additional normal all-female-producing strains have appeared.

As suggested above the various mothers in the sex intergrade strain differ markedly in the proportion of normal sex forms and sex intergrades while they produce. For example, for a given mother the percentage of males—normal males and definitely male intergrades of all grades—varies from 8 or 10% to 100%, the more usual figures being 45 to 85%. The percentages from the same mother from brood to brood vary of course but the sex array and its proportion in the first brood provide a basis for an estimate of what is to be expected in future broods.

The main interest in this material would seem to lie in the remarkable array of sex forms, the stock in general consisting of perhaps 40% normal males and about 8% normal females, the remainder being intergrades with *almost* every combination of male and female secondary and primary sex characters. More than half the individuals are neither wholly male nor wholly female but possess definite morphological sex characters of both sexes. The writer has attempted to classify these individuals on the basis of sex characters into more than twenty classes or grades. A grade at one end of the scale contains normal females; at the other end of the scale there is a class of normal males. Between these extreme grades are others for the many types of sex intergrades. Even this large number of grades is scarcely sufficient; many individuals occur which are intermediate between these grades and can with difficulty be assigned to any of them.

Sex here reveals itself not as a fixed and definite state but as a purely relative thing. No arbitrary classification into males and females is justifiable or possible; not only because of the confusing admixture of male and female secondary sex characters but also because the same individual, even the same sex gland, may develop eggs and sperm at the same time or sperm at one time and eggs at another time.

It seems probable that in this species sex depends upon a number of elements (I believe them to be largely or exclusively environmental factors) which influence the general physiological whole. In the stock which produces the sex intergrades the more usual association and the culmination of these influences has become interrupted. There is a consequent abnormal physiological balance resulting in haphazard combinations of sex-characters and a shifting of the balance one way or the other. Sometimes the result is a balance so unstable that in certain cases it seems to be shifted within a few days in the same individual or within portions of that individual as indicated by changes in the sex products.

Of secondary interest is the method of origin of the sex intergrade strain. The usual sex balance has become disturbed. The cause or method of this disturbance is not definitely known. But whatever the cause, the disturbance influenced not only the individual but also its germ plasm and the disturbed balance is evident throughout succeeding generations. The origin of this disturbance of the sexual balance may be referred to as a mutation. While there is little evidence concerning its cause, there seems abundant evidence concerning its permanent character so far as this strain is concerned.

The derivation from this sex intergrade strain of several strains which produce only *normal* (so far as may be judged by their morphological characters at any rate) females, and, on occasion, *normal* males, is a phenomenon similar to that of the sudden appearance of the sex intergrade strain and might with equal propriety be called a return mutation.

I am inclined to believe from evidence from many Cladocera, and from other forms reproducing parthenogenetically during most of the time and by means of sexual reproduction at irregular and uncertain intervals, that environmental factors in all such forms wield the determining influence. The evidence at hand in the present case, however, is not very conclusive and must be reserved for the larger paper.

[1] The following references may be cited:

Goldschmidt, R., Erblichkeitsstudien an Schmetterlingen. I, *Zs. ind. Abs.—Vererbungslehre*, **7**, 1-62 (1912); and a preliminary report on further experiments in inheritance and determination of sex, these PROCEEDINGS, **2**, 53–58.

Riddle, Oscar, Statement run in the *Carnegie Inst. Washington Year Book*, **12**, 322 (1913); and Sex control and known correlations in pigeons, *Amer. Nat.*, **50**, 385–410 (1916).

SOME PROBLEMS OF DIOPHANTINE APPROXIMATION: A REMARKABLE TRIGONOMETRICAL SERIES

By G. H. Hardy and J. E. Littlewood

TRINITY COLLEGE, CAMBRIDGE, ENGLAND

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1. The title of this note is perhaps not very appropriate: we retain it because the contents of the note form a natural sequel to those of three papers which we have published under same title elsewhere,[1] and in particular those of our second paper in the *Acta Mathematica.* We there discussed in detail the series

$$\sum e^{\alpha\pi i n^2 + 2\theta\pi i n}, \tag{1.1}$$

and other similar series associated with the elliptic Theta-functions, and used our results to elucidate a variety of difficult points in the theory